Discrete Structures

Fall 2015

Homework3

Chapter 5

1. **Section 5.1** **page 329**

   Problems: 3,5,7,9,11,15

3. Let $P(n)$ be the statement that $1^2 + 2^2 + \cdots + n^2 = n(n + 1)(2n + 1)/6$ for the positive integer n.

a) What is the statement $P(1)$?

b) Show that $P(1)$ is true, completing the basis step of the proof.

c) What is the inductive hypothesis?

d) What do you need to prove in the inductive step?

e) Complete the inductive step, identifying where you use the inductive hypothesis.

f) Explain why these steps show that this formula is true whenever n is a positive integer.

5. Prove that $1^2 + 3^2 + 5^2 + \cdots + (2n + 1)^2 = (n + 1)(2n + 1)(2n + 3)/3$ whenever n is a nonnegative integer.

7. Prove that $3+3 \cdot 5+3 \cdot 5^2+ \cdot \cdot \cdot + 3 \cdot 5^n=3(5^{n+1}-1)/4$ whenever n is a nonnegative integer.

9. **a)** Find a formula for the sum of the first n even positive integers.
b) Prove the formula that you conjectured in part (a).

1. **Section 5.3** **page 357**

Problems: 5,7,9,11

5. Determine whether each of these proposed definitions is a valid recursive definition of a function f from the set of nonnegative integers to the set of integers. If f is well defined, find a formula for f (n) when n is a nonnegative integer and prove that your formula is valid.

a) $f(0) = 0$, $f(n) = 2f(n-2)$ for $n \geq 1$

b) $f(0) = 1$, $f(n) = f(n-1) - 1$ for $n \geq 1$

c) $f(0) = 2$, $f(1) = 3$, $f(n) = f(n-1) - 1$ for $n \geq 2$

d) $f(0) = 1$, $f(1) = 2$, $f(n) = 2f(n-2)$ for $n \geq 2$

e) $f(0) = 1$, $f(n) = 3f(n-1)$ if n is odd and $n \geq 1$ and $f(n) = 9f(n-2)$ if n is even and $n \geq 2$

7. Give a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \ldots$ if

a) $a_n = 6n$.

b) $a_n = 2n + 1$.

c) $a_n = 10^n$.

d) $a_n = 5$.

9. Let F be the function such that $F(n)$ is the sum of the first n positive integers. Give a recursive definition of $F(n)$.

11. Give a recursive definition of $P_m(n)$, the product of the integer m and the nonnegative integer n.

Chapter 6

1. Section6.1 page 396

Problems: 7,9,11,13,15,21,23,33,53

7. How many different three-letter initials can people have?

9. How many different three-letter initials are there that begin with an *A*?

11. How many bit strings of length ten both begin and end with a 1?

13. How many bit strings with length not exceeding *n*, where *n* is a positive integer, consist entirely of 1s, not counting the empty string?

15. How many strings are there of lowercase letters of length four or less, not counting the empty string?

21. How many positive integers between 50 and 100

a) are divisible by 7? Which integers are these?

b) are divisible by 11? Which integers are these?

c) are divisible by both 7 and 11? Which integers are these?

23. How many positive integers between 100 and 999 inclusive

a) are divisible by 7?

b) are odd?

c) have the same three decimal digits?

d) are not divisible by 4?

e) are divisible by 3 or 4?
f) **F)** are not divisible by either 3 or 4?

g) are divisible by 3 but not by 4?

h) are divisible by 3 and 4?

33. How many strings of eight English letters are there

a) that contain no vowels, if letters can be repeated?
b) that contain no vowels, if letters cannot be repeated?
c) that start with a vowel, if letters can be repeated?
d) that start with a vowel, if letters cannot be repeated?
e) that contain at least one vowel, if letters can be repeated?
f) that contain exactly one vowel, if letters can be repeated?
g) that start with X and contain at least one vowel, if letters can be repeated?
h) that start and end with X and contain at least one vowel, if letters can be repeated?

53. How many positive integers not exceeding 100 are divisible either by 4 or by 6?

2. Section 6.2 page 405
Problems: 1,5,7,9,21,23

1. Show that in any set of six classes, each meeting regularly once a week on a particular day of the week, there must be two that meet on the same day, assuming that no classes are held on weekends.

5. Show that among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4.

7. Let n be a positive integer. Show that in any set of n consecutive integers there is exactly one divisible by n.

9. What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state?

21. Construct a sequence of 16 positive integers that has no increasing or decreasing subsequence of five terms.

***23.** Show that whenever 25 girls and 25 boys are seated around a circular table there is always a person both of whose neighbors are boys.

3. Section6.3 page 413

Problems: 3,5,7,9,11,13,15,17,27,33,37

3. How many permutations of {*a, b, c, d, e, f, g*} end with *a*?

5. Find the value of each of these quantities.

a) $P(6, 3)$

b) $P(6, 5)$

c) $P(8, 1)$

e) $P(8, 5)$

e) $P(8, 8)$

f) $P(10, 9)$

7. Find the number of 5-permutations of a set with nine elements.

9. How many possibilities are there for the win, place, and show (first, second, and third) positions in a horse race with 12 horses if all orders of finish are possible?

11. How many bit strings of length 10 contain

a) exactly four 1s?
b) at most four 1s?
c) at least four 1s?
d) an equal number of 0s and 1s?

13. A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate?

15. In how many ways can a set of five letters be selected from the English alphabet?

17. How many subsets with more than two elements does a set with 100 elements have?

27. A club has 25 members.

a) How many ways are there to choose four members of the club to serve on an executive committee?

b) How many ways are there to choose a president, vice president, secretary, and treasurer of the club, where no person can hold more than one office?

33. Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have the same number of men and women?

37. How many bit strings of length 10 contain at least three 1s and at least three 0s?

4. Section 6.4 page 421

Problems: 1,3,5,9,13

1. Find the expansion of $(x + y)^4$

a) using combinatorial reasoning, as in Example 1.

b) using the binomial theorem.

3. Find the expansion of $(x + y)^6$.

5. How many terms are there in the expansion of $(x + y)^{100}$ after like terms are collected?

9. What is the coefficient of $x^{101}y^{99}$ in the expansion of $(2x - 3y)^{200}$?

13. What is the row of Pascal’s triangle containing the binomial coefficients $\binom{9}{k}$, $0 \leq k \leq 9$?